Electromagnetic form factors of the nucleon in the chiral constituent quark model

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XIX International Workshop on Deep-Inelastic Scattering and Related Subjects (DIS 2011)
Jefferson Lab, Newport News, Virginia
April 11-15, 2011

Outline

Naive Quark Model

- **Internal Structure:** The knowledge of internal structure of nucleon in terms of quark and gluon degrees of freedom in QCD provides a basis for understanding more complex, strongly interacting matter.
- Knowledge has been rather limited because of confinement and it is still a big challenge to perform the calculations from the first principles of QCD.
- **Naive Quark Model** is able to provide a intuitive picture and successfully accounts for many of the low-energy properties of the hadrons in terms of the valence quarks.

Proton Spin Problem: The driving question

- 1988 European Muon Collaboration (Valence quarks carry 30% of proton spin)
- Naive Quark Model contradicts this results (Based on Pure valence description: proton = 2u + d)
  "Proton spin crisis"
- Confirmed by the measurements of polarized structure functions of proton in the deep inelastic scattering (DIS) experiments by SMC, E142-3 and HERMES experiments.
- Provides evidence that the valence quarks of proton carry only a small fraction of its spin suggesting that they should be surrounded by an indistinct sea of quark-antiquark pairs.

Flavor Structure

- 1991 NMC result: Asymmetric nucleon sea ($\bar{d} > \bar{u}$)
  Recently confirmed by E866 and HERMES
- Sum Rules
  - Bjorken Sum Rule: $\Delta_3 = \Delta u - \Delta d$
  - Ellis-Jaffe Sum Rule: $\Delta_8 = \Delta u + \Delta d - 2\Delta s$
    (Reduces to $\Delta_8 = \Delta\Sigma$ when $\Delta s = 0$)
  - Strange quark fraction: $f_s \simeq 0.10$
  - Gottfried Sum Rule: $I_G = \frac{1}{3} + \frac{2}{3}\int_0^1 [\bar{u}(x) - \bar{d}(x)]dx = 0.254 \pm 0.026$

Fundamental quantities

- **Structure:** Magnetic moments
  Dirac theory (1.0 μ_N) and experiment (2.5 μ_N).
  Proton is not an elementary Dirac particle but has an inner structure.
- **Size:** Spatial extension
  Proton charge distribution given by charge radius r_p.
- **Shape:** Nonspherical charge distribution
  Quadrupole moment of the transition $N \rightarrow \Delta$.
- Relation between the properties??

Non-perturbative regime

- Recently, a wide variety of accurately measured data have been accumulated for
  static properties of hadrons: masses, electromagnetic moments, charge radii etc.
  low energy dynamical properties: scattering lengths and decay rates etc.
- These lie in the non perturbative range of QCD.
- The direct calculations of these quantities form the first principle of QCD are extremely difficult, because they require non-perturbative methods.
- Techniques such as lattice gauge theory, QCD sum rules, and a wide variety of models have been developed to study this extremely interesting energy regime.

Chiral Constituent Quark Model

- χCQM initiated by Weinberg and developed by Manohar and Georgi to explain the successes of NQM.
- "Quark sea" generation $q_{\pm} \rightarrow GB^0 + q'_{\mp} \rightarrow (q\bar{q}') + q'_{\mp}$
- Incorporates *confinement* and *chiral symmetry breaking.*
- “Justifies” the idea of constituent quarks and scope of the model extended in the context of **“proton spin crisis”.**

Methodology

- "Quark sea" generation $q_{\pm} \rightarrow GB^0 + q'_{\mp} \rightarrow (q\bar{q}') + q'_{\mp}$
- $\mathcal{L} = g_8 \bar{q} \Phi q + g_1 \bar{q} \frac{\eta'}{\sqrt{3}} q = g_8 \bar{q} \left(\Phi + \zeta \frac{\eta'}{\sqrt{3}} I \right) q$
- $\Phi' =$
$$\begin{pmatrix} \phi_{uu} u\bar{u} + \phi_{ud} d\bar{d} + \phi_{us} s\bar{s} & \varphi_{ud} u\bar{d} & \varphi_{us} u\bar{s} \\ \varphi_{du} d\bar{u} & \phi_{du} u\bar{u} + \phi_{dd} d\bar{d} + \phi_{ds} s\bar{s} & \varphi_{ds} d\bar{s} \\ \varphi_{su} s\bar{u} & \phi_{sd} s\bar{d} & \phi_{su} u\bar{u} + \phi_{sd} d\bar{d} + \phi_{ss} s\bar{s} \end{pmatrix}$$

Transition probabilities

- $$\begin{aligned}
\phi_{uu} &= \phi_{dd} = \frac{1}{2} + \frac{\beta}{6} + \frac{\zeta}{3}\,, \quad \phi_{ss} = \frac{2\beta}{3} + \frac{\zeta}{3}\,, \\
\phi_{us} &= \phi_{ds} = \phi_{su} = \phi_{sd} = -\frac{\beta}{3} + \frac{\zeta}{3}\,, \quad \phi_{du} = \phi_{ud} = -\frac{1}{2} + \frac{\beta}{6} + \frac{\zeta}{3}\,, \\
\varphi_{ud} &= \varphi_{du} = 1\,, \quad \varphi_{us} = \varphi_{ds} = \varphi_{su} = \varphi_{sd} = \alpha\,.
\end{aligned}$$
- The parameter $a(= |g_8|^2)$ denotes the transition probability of chiral fluctuation of the splittings $u(d) \rightarrow d(u) + \pi^{+(-)}$, whereas $\alpha^2 a$, $\beta^2 a$ and $\zeta^2 a$ respectively denote the probabilities of transitions of $u(d) \rightarrow s + K^{-(o)}$, $u(d,s) \rightarrow u(d,s) + \eta$, and $u(d,s) \rightarrow u(d,s) + \eta^{'}$.

Importance

- The electromagnetic form factors are fundamental quantities of theoretical and experimental interest to describe the internal structure of nucleon.
- The measurements of nucleon form factors at low momentum transfer are sensitive to the pion cloud and provide a test for the nucleon models and effective field theories of the QCD based on the chiral symmetry.
- The knowledge of internal structure of nucleon in terms of quarks and gluons degrees of freedom of QCD provide a basis for understanding more complex, strongly interacting matter at the level of quarks and gluons.

Dirac and Pauli form factors

- Hadronic current for a spin $\frac{1}{2}$-nucleon with internal structure is
  $\langle B|J^{\mu}_{\rm had}(0)|B\rangle = \bar{u}(p')\left(\gamma^{\mu}F_1(Q^2) + i\frac{\sigma^{\mu\nu}}{2M}q_{\nu}F_2(Q^2)\right)u(p)$
  $u(p)$ and $u(p')$ are the 4-spinors of the nucleon in the initial and final states.
- The Dirac and Pauli form factors $F_1(Q^2)$ and $F_2(Q^2)$ are the only two form factors allowed by relativistic invariance.
- They are normalized in such a way that at $Q^2 = 0$, they reduces to electric charge and the anomalous magnetic moment in units of the elementary charge and the nuclear magneton $e/(2m_p)$
  $F_1^p(0) = 1, F_2^p(0) = \kappa_p = 1.793$
  $F_1^n(0) = 0, F_2^n(0) = \kappa_n = -1.913.$

Charge and Magnetization densities

- Elastic *ep* scattering experiments provide detailed information on the radial variation of the charge and magnetization densities.
- We have $J_\mu = \left(G_E(Q^2)\ ,\ i\frac{\vec{\sigma}\times\vec{q}}{2M} G_M(Q^2) \right)$
  $G_E(Q^2)$: Time-like component of J_μ (Fourier transform of the electric charge distribution).
  $G_M(Q^2)$: Fourier transform of the magnetization density.
- The Sachs form factors G_E and G_M are related to the Dirac form factors by
  $G_E^N(Q^2) = F_1^N(Q^2) - \tau F_2^N(Q^2)$
  $G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2)$

- In non relativistic limit in Breit frame, the three-dimensional Fourier transform of $G_E(Q^2)$ provides the electric-charge-density distribution within the nucleon
$G_E(\vec{q}^{\,2}) = \int \rho_E(\vec{r}) e^{i\vec{q}\cdot\vec{r}} d^3\vec{r} = \int \rho_E(\vec{r}) d^3\vec{r} - \frac{\vec{q}^{\,2}}{6} \int \rho_E(\vec{r}) \vec{r}^{\,2} d^3\vec{r} + \ ...$
- The first integral yields the total charge in units of e, i.e., 1 for the proton and 0 for the neutron, and the second integral defines the square of the electric root-mean-square (rms) radius, $\langle r^2 \rangle_E = -6 \frac{d}{dQ^2} G_E(Q^2)|_{Q^2=0}$.
- $G_M(Q^2)$ gives the magnetic-current-density distribution
$G_M(\vec{q}^{\,2}) = \int \rho_M(\vec{r}) e^{i\vec{q}\cdot\vec{r}} d^3\vec{r}$
$\mu_B = G_M(0)$

Magnetic moment

- The magnetic moment is defined as the expectation value of the z component of magnetic moment operator with maximal projection along the $z-$axis
  $\mu(B) = \langle B|\mu_z|B\rangle = \sum_{u,d,s}\langle B|\mu_q\sigma_z^q|B\rangle$,
  where $\mu_q = \frac{e_q}{2M_q}$ is the quark magnetic moments
- In a simplified manner, the magnetic moment of a baryon can be expressed as the sum of its constituent quark's Dirac moment
  $\mu(B) = \sum_{q=u,d,s} \Delta q^B \mu_q = \Delta u^B \mu_u + \Delta d^B \mu_d + \Delta s^B \mu_s$.

Magnetic Moment in χCQM

- Magnetic moment of a given baryon receive contributions from the valence quarks, "quark sea" and orbital angular momentum of the "quark sea" as $\mu(B)_{\rm total} = \mu(B)_{\rm val} + \mu(B)_{\rm sea} + \mu(B)_{\rm orbit}$.
- In terms of quarks magnetic moments and spin polarizations
  $\mu(B)_{\rm val} = \sum_{q=u,d,s} \Delta q_{\rm val} \mu_q$,
  $\mu(B)_{\rm sea} = \sum_{q=u,d,s} \Delta q_{\rm sea} \mu_q$,
  $\mu(B)_{\rm orbit} = \sum_{q=u,d,s} \Delta q_{\rm val}\ \mu(q_+ \rightarrow)$.
- $\mu(q_+ \rightarrow)$ is the orbital moment for any chiral fluctuation ,
  e_q and M_q are the electric charge and the mass for quark q.

Spin structure and magnetic moment

- $\widehat{B} = \langle B, J_z = \frac{1}{2} | \mathcal{N} | B, J_z = \frac{1}{2} \rangle$
  $\mathcal{N} = n_{u_+} u_+ + n_{u_-} u_- + n_{d_+} d_+ + n_{d_-} d_- + n_{s_+} s_+ + n_{s_-} s_-$,
- Valence quark spin polarizations $\Delta q_{\rm val} = n_{q_+} - n_{q_-}$.
- For "quark sea" contribution substitute for each valence quark by $q_\pm \to -\sum P_q q_\pm + |\psi(q_\pm)|^2$, where $\sum P_q$ is the probability of emission of GBs from a q quark and $|\psi(q_\pm)|^2$ is the probability of transforming a $q_\pm$ quark.
- The orbital angular momentum contribution of each chiral fluctuation
  $\mu(q_+ \to q'_-) = \frac{e_{q'}}{2M_q} \langle l_q \rangle + \frac{e_q - e_{q'}}{2M_{\rm GB}} \langle l_{\rm GB} \rangle$,
  where $\langle l_q \rangle = \frac{M_{\rm GB}}{M_q + M_{\rm GB}}$ and $\langle l_{\rm GB} \rangle = \frac{M_q}{M_q + M_{\rm GB}}$
  The quantities (l_q, $l_{\rm GB}$) and (M_q, $M_{\rm GB}$) are the orbital angular momenta and masses of quark and GBs, respectively.

Baryon SU(6) wavefunctions

- The octet baryon wavefunction

$$|B\rangle \equiv \left|8, \frac{1}{2}^{+}\right\rangle = \frac{1}{\sqrt{2}}(\varphi^{'}\chi^{'} + \varphi^{''}\chi^{''})\psi^{s}(0^{+})$$

- The decuplet baryon wavefunction

$$|B^{*}\rangle \equiv |10, \frac{3}{2}^{+}\rangle = \chi^{s}\varphi^{s}\psi^{s}(0^{+})$$

χ, φ and ψ are the spin, isospin and spatial wavefunctions.

- **Input Parameters** χCQM involve the symmetry breaking parameters following the hierarchy $a > a\alpha^2 > a\beta^2 > a\zeta^2$.
  A fine grained analysis leads to
  $a = 0.12\,, \quad \alpha \simeq \beta = 0.45\,, \quad \text{and} \quad \zeta = -0.15\,.$
  For evaluating the quark and GB mass contributions, we have used their on shell mass values.

Magnetic moments of spin $\frac{1}{2}^+$ low lying baryons

Baryon	Data	NQM	Valence	Sea	Orbital	Total
$\mu(p)$	2.79±0.00	3	2.90	−0.58	0.48	2.80
$\mu(n)$	−1.91±0.00	−2	−1.85	0.18	−0.44	−2.11
$\mu(\Sigma^+)$	2.46±0.01	2.88	2.50	−0.51	0.40	2.39
$\mu(\Sigma^0)$	...	0.88	0.74	−0.22	0.02	0.54
$\mu(\Sigma^-)$	−1.16 ±0.025	−1.12	−1.02	0.06	−0.36	−1.32
$\mu(\Xi^0)$	−1.25 ±0.014	− 1.53	−1.29	0.14	−0.09	−1.24
$\mu(\Xi^-)$	−0.651±0.003	−0.53	−0.59	0.03	0.06	−0.50
ΔCG	0.49± 0.05	0.0	0.53	−0.08	0.01	0.46
$\mu(\Lambda)$	−0.613±0.004	−0.65	−0.59	0.02	−0.01	−0.58

Magnetic moments of spin $\frac{1}{2}^+$ low lying baryons

- Model is able to get a fairly good account of the most of magnetic moments.
- Magnetic moments of p, Σ^+, Ξ^0, and Λ give a perfect fit whereas for all other octet baryons our predictions are within 10% of the observed values.
- Excellent fit to ΔCG, the fit becomes more impressive when it is realized that none of the magnetic moments are used as inputs and ΔCG can be described without resorting to additional parameters.
- Sea and orbital contributions are fairly significant as compared to the valence contributions and they cancel in the right direction giving the right magnitude of the total magnetic moment.

Magnetic moments of spin $\frac{3}{2}^+$ low lying baryons

Baryon	Data	NQM	Valence	Sea	Orbital	Total
$\mu(\Delta^{++})$	$3.7 \sim 7.5$	6	4.53	-0.97	0.95	4.51
$\mu(\Delta^{+})$	$2.7^{+1.0}_{-1.3} \pm 1.5 \pm 3$	3	2.27	-0.61	0.34	2.00
$\mu(\Delta^{0})$	...	0.0	0.06 ± 0.0	-0.25	-0.26	-0.51
$\mu(\Delta^{-})$	...	-3	-2.27	0.12	-0.87	-3.02
$\mu(\Sigma^{*+})$	...	3.35	2.74	-0.67	0.62	2.69
$\mu(\Sigma^{*0})$	...	0.35	0.29	-0.29	0.02	0.02
$\mu(\Sigma^{*-})$	...	-2.65	-2.16	0.11	-0.59	-2.64
$\mu(\Xi^{*0})$	...	0.71	0.51	-0.26	0.29	0.54
$\mu(\Xi^{*-})$	...	-2.29	-1.64	0.08	-0.31	-1.87
$\mu(\Omega^{-})$	-2.02 ± 0.06	-1.94	-1.76	0.08	-0.03	-1.71
	-1.94 ± 0.31					

Magnetic moments of spin $\frac{3}{2}^+$ low lying baryons

- The sea and orbital contributions to the magnetic moments are significant. The orbital part contributes with the same sign as valence part whereas the sea part contributes with the opposite sign. They cancel each other in the right order.
- The residual contribution along with the valence contribution give the magnetic moment of baryons of the correct order.
- Results obtained are consistent with the Light Cone QCD Sum Rules and also with other models.

Charge Radius

- The charge radius operator can be expressed as a sum of one-, two-, and three-quark terms in spin-flavor space as
  $r^2 = A\sum_{i=1}^{3} e_i \mathbf{1} + B\sum_{i\neq j}^{3} e_i\,\sigma_{\mathbf{i}}\cdot\sigma_{\mathbf{j}} + \mathbf{C}\sum_{\mathbf{i}\neq\mathbf{j}\neq\mathbf{k}}^{\mathbf{3}} \mathbf{e_k}\,\sigma_{\mathbf{i}}\cdot\sigma_{\mathbf{j}}$
- $e_i = (1+3\tau_{i\,z})/6$ and $\sigma_{\mathbf{i}}$ are the charge and spin of the i-th quark. $\tau_{i\,z}$ denotes the z component of the Pauli isospin matrix.
  The constants A, B, and C parameterizing the orbital and color matrix elements are determined from experiment.
- The matrix element can be calculated for the charge radius operator and the wavefunction for the baryon state.

Quadrupole Moment

- Similarly, the charge quadrupole operator is composed of a two- and three-body term in spin-flavor space
  $\mathcal{Q} = B' \sum_{i \neq j}^{3} e_i \left(3\sigma_{iz}\sigma_{jz} - \sigma_{\mathbf{i}} \cdot \sigma_{\mathbf{j}}\right) + C' \sum_{i \neq j \neq k}^{3} e_k \left(3\sigma_{iz}\sigma_{jz} - \sigma_{\mathbf{i}} \cdot \sigma_{\mathbf{j}}\right)$.
- Baryon decuplet quadrupole moments Q_{B^*} and octet-decuplet transition quadrupole moments $Q_{B \to B^*}$ are obtained by calculating the matrix elements of the quadrupole operator between the three-quark spin-flavor wave functions $Q_{B^*} = \langle B^* | \mathcal{Q} | B^* \rangle$
  $Q_{B \to B^*} = \langle B^* | \mathcal{Q} | B \rangle$
- B denotes a spin 1/2 octet baryon and B^* a member of the spin 3/2 baryon decuplet.

Charge radii and quadrupole moment operators

- Operators simplified to find the expectation values

$$\begin{aligned}
\sum_{i\neq j} e_i(\sigma_{\mathbf{i}}\cdot\sigma_{\mathbf{j}}) &= 2\mathbf{J}\cdot\sum_{\mathbf{i}}\mathbf{e_i}\sigma_{\mathbf{i}} - \mathbf{3}\sum_{\mathbf{i}}\mathbf{e_i} \\
\sum_{i\neq j\neq k} e_i(\sigma_{\mathbf{j}}\cdot\sigma_{\mathbf{k}}) &= \pm 3\sum_i e_i - \sum_{i\neq j} e_i(\sigma_{\mathbf{i}}\cdot\sigma_{\mathbf{j}})
\end{aligned}$$

- $+$ sign holds for $J=\frac{3}{2}$ and $-$ sign for $J=\frac{1}{2}$ states.
- Rewriting

$$\begin{aligned}
\textstyle\sum_{i\neq j\neq k} e_i(\sigma_{\mathbf{j}}\cdot\sigma_{\mathbf{k}}) &= -2\mathbf{J}\cdot\textstyle\sum_{\mathbf{i}}\mathbf{e_i}\sigma_{\mathbf{i}} && \left(\text{for } J=\tfrac{1}{2}\right) \\
&= 6\textstyle\sum_i e_i - 2\mathbf{J}\cdot\sum_{\mathbf{i}}\mathbf{e_i}\sigma_{\mathbf{i}} && \left(\text{for } J=\tfrac{3}{2}\right)
\end{aligned}$$

Charge radii and quadrupole moment operators

- The expectation value of operator $2J \cdot \sum_i e_i \sigma_{\mathbf{i}}$ between the baryon wavefunctions $|B\rangle$ in the initial and final states can easily be calculated.
- $$\begin{aligned} \sum_{i \neq j} e_i (\sigma_{\mathbf{i}} \cdot \sigma_{\mathbf{j}}) &= 3 \sum_i e_i \sigma_{\mathbf{iz}} - \mathbf{3} \sum_{\mathbf{i}} \mathbf{e_i} && \left(\text{for } J = \tfrac{1}{2}\right) \\ &= 5 \sum_i e_i \sigma_{\mathbf{iz}} - \mathbf{3} \sum_{\mathbf{i}} \mathbf{e_i} && \left(\text{for } J = \tfrac{3}{2}\right) \end{aligned}$$
- $$\begin{aligned} \sum_{i \neq j \neq k} e_i (\sigma_{\mathbf{j}} \cdot \sigma_{\mathbf{k}}) &= -3 \sum_i e_i \sigma_{\mathbf{iz}} && \left(\text{for } J = \tfrac{1}{2}\right) \\ &= 6 \sum_i e_i - 5 \sum_i e_i \sigma_{\mathbf{iz}} && \left(\text{for } J = \tfrac{3}{2}\right) \end{aligned}$$

Charge radii and quadrupole moment operators

- The expression for the charge radii for the spin $\frac{1}{2}^+$ and spin $\frac{3}{2}^+$ baryons can be expressed as

$$\begin{aligned} r^2_{1/2} &= (\mathrm{A} - 3\mathrm{B}) \sum_i e_i + 3(\mathrm{B} - \mathrm{C}) \sum_i e_i \sigma_{iz} \\ r^2_{3/2} &= (\mathrm{A} - 3\mathrm{B} + 6\mathrm{C}) \sum_i e_i + (5\mathrm{B} - 5\mathrm{C}) \sum_i e_i \sigma_{\mathbf{iz}} \end{aligned}$$

- Similarly, the expression for the quadrupole moment for the spin $\frac{1}{2}^+$ and spin $\frac{3}{2}^+$ baryons can be expressed as

$$\begin{aligned} \mathcal{Q}_{1/2} &= \mathrm{B}' \left(3 \sum_{i \neq j} e_i \sigma_{\mathbf{iz}} \sigma_{\mathbf{jz}} - \mathbf{3} \sum_{\mathbf{i}} \mathbf{e_i} \sigma_{\mathbf{iz}} + \mathbf{3} \sum_{\mathbf{i}} \mathbf{e_i} \right) + \mathrm{C}' \left(3 \sum_{i \neq j \neq k} e_i \sigma_{\mathbf{jz}} \sigma_{\mathbf{kz}} + \mathbf{3} \sum_{\mathbf{i}} \mathbf{e_i} \sigma_{\mathbf{iz}} \right) \\ \mathcal{Q}_{3/2} &= \mathrm{B}' \left(3 \sum_{i \neq j} e_i \sigma_{\mathbf{iz}} \sigma_{\mathbf{jz}} - \mathbf{5} \sum_{\mathbf{i}} \mathbf{e_i} \sigma_{\mathbf{iz}} + \mathbf{3} \sum_{\mathbf{i}} \mathbf{e_i} \right) + \mathrm{C}' \left(3 \sum_{i \neq j \neq k} e_i \sigma_{\mathbf{jz}} \sigma_{\mathbf{kz}} - \mathbf{6} \sum_{\mathbf{i}} \mathbf{e_i} + \mathbf{5} \sum_{\mathbf{i}} \mathbf{e_i} \sigma_{\mathbf{iz}} \right) \end{aligned}$$

Flavor expectation value

- The calculation of charge radii and quadrupole moment reduces to the calculation of the spin term ($\sum_i e_i \sigma_i$), flavor term ($\sum_i e_i$) and the tensor terms ($\sum_i e_i \sigma_{iz}\sigma_{jz}$ and $\sum_i e_i \sigma_{jz}\sigma_{kz}$) for a given baryon.
- The flavor content ($\sum_i e_i$) of a given baryon can be calculated from the expectation value $\widehat{B} \equiv \langle B|\sum_i e_i|B\rangle$, where $|B\rangle$ is the baryon wave function and $\sum_i e_i$ is the number operator defined as

$$\sum_i e_i = \sum_{q=u,d,s} n_q^B q + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} n_{\bar{q}}^B \bar{q} = n_u^B u + n_d^B d + n_s^B s + n_{\bar{u}}^B \bar{u} + n_{\bar{d}}^B \bar{d} + n_{\bar{s}}^B \bar{s}$$

  n_q^B is the number of quarks with charge q.
- For a given baryon $q = -\bar{q}$ and

$$\sum_i e_i = (n_u^B - n_{\bar{u}}^B)u + (n_d^B - n_{\bar{d}}^B)d + (n_s^B - n_{\bar{s}}^B)s$$

Spin expectation value

- The spin structure ($\sum_i e_i\sigma_i$) of the baryon is defined as $\widehat{B} \equiv \langle B| \sum_i e_i\sigma_i |B\rangle$, with

$$\sum_i e_i\sigma_i = \sum_{q=u,d,s} n^B_{q_\pm} q_\pm = n^B_{u_+} u_+ + n^B_{u_-} u_- + n^B_{d_+} d_+ + n^B_{d_-} d_- + n^B_{s_+} s_+ + n^B_{s_-} s_-$$

$n^B_{q_\pm}$ being the number of $q_\pm$ quarks.

- For a given baryon $q_+ = -q_-$ and

$$\begin{aligned} \sum_i e_i\sigma_i &= (n^B_{u_+} - n^B_{u_-})u_+ + (n^B_{d_+} - n^B_{d_-})d_+ + (n^B_{s_+} - n^B_{s_-})s_+ \, , \\ &= \sum_{u,d,s} (\Delta q^B) q_+ , \end{aligned}$$

where $\Delta q^B = n^B_{q_+} - n^B_{q_-}$, being the spin polarizations function.

Charge radii of octet baryons in χCQM

Charge radii	Data	NQM	with SU(3) symmetry	with SU(3) symmetry breaking	
				$\mathrm{A} = 0.879$ $\mathrm{B} = 0.094$ $\mathrm{C} = 0.0$	$\mathrm{A} = 0.879$ $\mathrm{B} = 0.094$ $\mathrm{C} = 0.016$
r_p^2	r_p= 0.877$\pm$ 0.007	0.813	0.732	0.801	0.766
r_n^2	-0.1161 ± 0.0022	-0.138	-0.087	-0.140	-0.116
$r_{\Sigma^+}^2$	...	0.813	0.732	0.802	0.767
$r_{\Sigma^-}^2$	0.61 $\pm$0.21	0.675	0.646	0.678	0.664
$r_{\Sigma^0}^2$	...	0.069	0.043	0.062	0.052
$r_{\Xi^0}^2$	...	-0.138	-0.087	-0.145	-0.120
$r_{\Xi^-}^2$	...	0.675	0.646	0.683	0.669
r_{Λ}^2	...	-0.069	-0.042	-0.076	-0.063
$r_{\Sigma\Lambda}^2$	...	0.135	0.085	0.132	0.109

Charge radii of octet baryons in χCQM

- To understand the implications of chiral symmetry breaking and "quark sea", we have also presented the results of NQM including the one-, two-, and three-quark contributions of the GP parameters.
- SU(3) symmetry breaking corrections are of the order of 5% for the case of p, Σ^+, Σ^-, and Ξ^- baryons whereas this contribution is more than 20% for the neutral octet baryons.
- Inclusion of the three-quark term ($\mathrm{C} = 0.016$) decreases the value of the octet baryon charge radii which may be due to the spin of the "quark sea" contributing with an opposite sign.

Charge radii of decuplet baryons in χCQM

Charge radii	NQM	χCQM with SU(3)		
		symmetry	symmetry breaking	
			$A = 0.879$ $B = 0.094$ $C = 0.0$	$A = 0.879$ $B = 0.094$ $C = 0.016$
$r^2{}_{\Delta^{++}}$	1.084	0.938	0.961	0.996
$r^2{}_{\Delta^{+}}$	1.084	0.938	0.946	0.983
$r^2{}_{\Delta^{0}}$	0.0	0.0	-0.030	-0.025
$r^2{}_{\Delta^{-}}$	1.084	0.938	1.006	1.033
$r^2{}_{\Sigma^{*+}}$	1.084	0.938	0.940	0.978
$r^2{}_{\Sigma^{*-}}$	1.084	0.938	1.013	1.038
$r^2{}_{\Sigma^{*0}}$	0.0	0.0	-0.036	-0.030
$r^2{}_{\Xi^{*0}}$	0.0	0.0	-0.043	-0.035
$r^2{}_{\Xi^{*-}}$	1.084	0.938	1.019	1.043
$r^2{}_{\Omega^{-}}$	0.390	0.245	0.429	0.355

- The spin $\frac{3}{2}^+$ decuplet baryon charge radii are in general higher than the spin $\frac{1}{2}^+$ octet baryon charge radii which is in line with the trend followed for the other low energy hadronic matrix elements such as magnetic moments.
- SU(3) symmetry breaking increases the predictions of charge radii. It can be easily shown that SU(3) symmetry results in following relations for the decuplet baryons

$$r^2_{\Delta^{++}} = r^2_{\Delta^+} = r^2_{\Delta^-} = r^2_{\Sigma^{*+}} = r^2_{\Sigma^{*-}} = r^2_{\Xi^{*-}} \,.$$

- Inclusion of SU(3) symmetry breaking give

$$r^2_{\Xi^{*-}} > r^2_{\Sigma^{*-}} > r^2_{\Delta^-} > r^2_{\Delta^{++}} > r^2_{\Delta^+} > r^2_{\Sigma^{*+}} \,.$$

- Relations derived in $1/N_c$ expansion of QCD also holds

$$\begin{aligned} 2r^2_{\Delta^{++}} - r^2_{\Delta^+} - r^2_{\Delta^0} - r^2_{\Delta^-} &= 0\,, \\ 2r^2_{\Delta^{++}} - 3r^2_{\Delta^+} + 3r^2_{\Delta^0} + r^2_{\Delta^-} &= 0\,, \\ r^2_{\Sigma^{*+}} - 2r^2_{\Sigma^{*0}} - r^2_{\Sigma^{*-}} &= 0\,. \end{aligned}$$

Quadrupole moments of the spin $\frac{1}{2}^+$ octet and spin $\frac{3}{2}^+$ decuplet baryons.

Octet			Decuplet		
Baryon	NQM	χCQM	Baryon	NQM	χCQM
		$\mathrm{B}' = -0.047$ $\mathrm{C}' = -0.008$			$\mathrm{B}' = -0.047$ $\mathrm{C}' = -0.008$
			Δ^{++}	-0.409	-0.3695
p	0.0	-0.031	Δ^{+}	-0.204	-0.1820
n	0.0	-0.012	Δ^{0}	0.0	0.0055
			Δ^{-}	0.204	0.1930
Σ^{+}	0.0	-0.033	Σ^{*+}	-0.204	-0.1808
Σ^{-}	0.0	0.015	Σ^{*-}	0.204	0.1942
Σ^{0}	0.0	-0.009	Σ^{*0}	0.0	0.0067
Ξ^{0}	0.0	-0.015	Ξ^{*0}	0.0	0.0079
Ξ^{-}	0.0	0.014	Ξ^{*-}	0.204	0.1954
Λ	0.0	-0.012	Ω^{-}	0.204	0.1966

Quadrupole moments of the spin $\frac{3}{2}^+ \to \frac{1}{2}^+$ decuplet to octet transitions

Baryon	NQM	χCQM
		$\mathrm{B}' = -0.047$ $\mathrm{C}' = -0.008$
$\Delta^+ p$	-0.110	-0.0846
$\Sigma^{*+}\Sigma^+$	-0.110	-0.0864
$\Sigma^{*-}\Sigma^-$	0.0	-0.0018
$\Sigma^{*0}\Sigma^0$	-0.055	-0.0441
$\Xi^{*0}\Xi^0$	-0.110	-0.0864
$\Xi^{*-}\Xi^-$	0.0	-0.0018
$\Sigma^{*0}\Lambda$	-0.096	-0.0733

Application Potential of the model

The present calculations suggest few important points

- Decomposition of various measurable quantities into the contributions from valence and sea components.
- Contribution of strange quarks in the nucleon which do not appear explicitly in most quark model descriptions of the nucleon and the role played by non-valence flavors in understanding the nucleon internal structure.

Long term

Understanding the spin structure of the proton will help to resolve the most challenging problems facing subatomic physics which include

- What happens to the spin in the transition between current and constituent quarks in the low energy QCD?
- How can we distinguish between the *current quarks* and the *constituent quarks*?
- How is the spin of the proton built out from the intrinsic spin and orbital angular momentum of its quark and gluonic constituents?
- What is the role played by non-valence flavors in understanding the nucleon internal structure?

Summary and Conclusions

- A small but non-zero value of SU(3) symmetry breaking within the dynamics of χCQM, suggests an important role for non-valence quark masses in the non-perturbative regime of QCD.
- Substantiated by a measurement of the baryon charge radius and other transition quadrupole moments.
- At leading order, the model envisages constituent quarks, the Goldstone bosons (π, K, η mesons) as appropriate degrees of freedom